Inference at $*$ 1
of proof for Lemma p-mu-exists:

1. $P : \mathbb{N} \to \mathbb{B}$
2. $\exists n{:}\mathbb{N}.\ (\uparrow(P(n)))$

$\vdash \exists x{:}\mathbb{N} + \text{Top}.\ \text{p-mu}(P;x)$

```
  by ((ExRepD·)
  CollapseTHEN ((((if (((first_nat 2:n)) = 0) then (Repeat (
    MoveToConcl (-1))) else (RepeatFor (first_nat 2:n) (MoveToConcl (-1))))·)

      CollapseTHEN (((CompleteInductionOnNat)
  CollapseTHEN (((Auto·)
  CollapseTHEN (((
    Decide ∃i:{0..n⁻}. (↑(P(i)))
  ·)
CollapseTHENA (Auto·))·))·))·))·))·))·
```

1:

2. $n : \mathbb{N}$
3. $\forall n_1{:}\mathbb{N}.\ (n_1 < n) \Rightarrow (\uparrow(P(n_1))) \Rightarrow (\exists x{:}\mathbb{N} + \text{Top}.\ \text{p-mu}(P;x))$
4. $\uparrow(P(n))$
5. $\exists i{:}\{0..n^-\}.\ (\uparrow(P(i)))$

$\vdash \exists x{:}\mathbb{N} + \text{Top}.\ \text{p-mu}(P;x)$

2:

2. $n : \mathbb{N}$
3. $\forall n_1{:}\mathbb{N}.\ (n_1 < n) \Rightarrow (\uparrow(P(n_1))) \Rightarrow (\exists x{:}\mathbb{N} + \text{Top}.\ \text{p-mu}(P;x))$
4. $\uparrow(P(n))$
5. $\neg(\exists i{:}\{0..n^-\}.\ (\uparrow(P(i))))$

$\vdash \exists x{:}\mathbb{N} + \text{Top}.\ \text{p-mu}(P;x)$

.